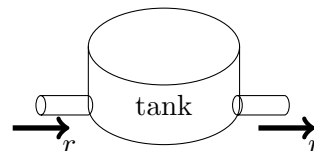


3. [14 points] Consider a storage tank containing  $V_0$  liters of pure water, having an open top, as suggested in the figure to the right. Water containing a chemical at a concentration of  $c_0$  kg/liter enters the tank at a rate  $r$  liters/min. The well-mixed solution leaves the tank at the same rate.



- a. [5 points] Write down an initial value problem for the amount  $A$  of the chemical  $P$  in the tank.

- b. [5 points] Now suppose that the liquid can evaporate from the top of the tank. This results in a loss proportional to the surface area, so the volume of liquid in the tank decreases by  $\alpha\pi a^2$  liters/min, where  $a$  is the radius of the (cylindrical) tank. Write (but do not solve) a new differential equation for the amount of chemical in the tank. You should assume that the chemical does not evaporate as well. Be sure that it is clear why your equation has the form it does.

- c. [4 points] Consider your differential equation in (b) with the initial condition  $A(0) = c_0V_0$ . On what range of  $t$  values, if any, is a unique solution for  $A$  guaranteed to exist? Explain.