

4. [15 points] In lab 2 we considered the van der Pol equation, $x'' + \mu(x^2 - 1)x' + x = 0$. We consider this equation in this problem.
- a. [4 points] Write the van der Pol equation as a system of two first-order differential equations and show that the only critical point of the system is $(0, 0)$.
- b. [6 points] Let $\mu = 1$. As we saw in lab, the linearization of the system you obtained in (a) at the critical point is then $\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x}$. Solve this system and sketch a phase portrait for it.
- c. [5 points] Explain what your solution in (b) tells about solutions to the original van der Pol equation. Then, using your system from (a), find the slope of the trajectory in the phase plane at $(3, -\frac{3}{8})$. Explain what these tell you about how the phase portrait for (a) is different from that for (b), and how this is related to your work in lab.