5. [15 points] The following considers the solution  $(x_1, x_2)$  to a linear system of two first-order constant coefficient equations,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

**a**. [5 points] If the solutions to this system for two different initial conditions are shown to the right (in both graphs, the solid curve is  $x_1$  and the dashed curve is  $x_2$ ), sketch the corresponding trajectories in the phase plane. Label each trajectory.



**b**. [5 points] Given your trajectories in (a), give possible values for the eigenvalues and eigenvectors of the matrix **A**. Be sure that it is clear how you obtain your answer.

**c.** [5 points] Sketch a phase portrait for the system given your answer to (b). (If you were unable to complete (b), assume that your eigenvalues and eigenvectors are  $\lambda = -2$  with  $\mathbf{v} = \begin{pmatrix} 1 & -1 \end{pmatrix}^T$  and  $\lambda = -1$  with  $\mathbf{v} = \begin{pmatrix} 2 & -1 \end{pmatrix}^T$ .)