

7. [12 points] Each of the following has an answer that you can determine with minimal work. In each,  $\mathbf{A}$  is a  $2 \times 2$  real-valued matrix (but in each is a different matrix). Provide the answer, and give a two sentence explanation of how you obtained it.

a. [4 points] If  $\mathbf{A} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and eigenvalues of  $\mathbf{A}$  are  $\lambda_1$  and  $\lambda_2$ , with corresponding eigenvectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$ , then the general solution to  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$  is

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b. [4 points] If the only eigenvalue of  $\mathbf{A}$  is  $\lambda = -3$ , with only one eigenvector,  $\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ , then as  $t \rightarrow \infty$ , the largest term in all solutions of  $\mathbf{x}' = \mathbf{A}\mathbf{x}$  will be

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c. [4 points] If the eigenvalues of  $\mathbf{A}$  are  $\lambda = -3$  and  $\lambda = 5$ , with eigenvectors  $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ , then the number of solutions  $\mathbf{x}$  to  $\mathbf{A}\mathbf{x} = \mathbf{0}$  is

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and the number of solutions to  $\mathbf{A}\mathbf{x} = 3\mathbf{x}$  is

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