1. [15 points] Solve each of the following, finding explicit real-valued solutions as indicated.

a. [8 points] Find the solution to the initial value problem $\dfrac{y'}{x^3+y} = \dfrac{1}{x}$, $y(1) = 2$.

Solution: This is a linear problem, in standard form $y' - \frac{1}{x}\,y = x^2$. The integrating factor is $\mu = e^{-\int \frac{1}{x}\,dx} = x^{-1}$, so after multiplying through by μ we have $(\frac{1}{x}\,y)' = x$. Integrating and multiplying through by x gives $y = \frac{1}{2}x^3 + Cx$, so that with $y(1) = 2$ we have $C = \frac{3}{2}$, and

$$y = \frac{1}{2}x^3 + \frac{3}{2}x.$$

b. [7 points] Find the general solution to $y' + \frac{1}{t}\,y = \frac{1}{ty}$.

Solution: This is not linear, but can be separated. We have $y' = \frac{1}{t}(\frac{1}{y} - y)$, so that $\dfrac{y'}{\frac{1}{y} - y} = \dfrac{1}{t}$, or $\dfrac{yy'}{1-y^2} = \dfrac{1}{t}$. We are able to integrate both sides, finding $-\frac{1}{2}\ln|1-y^2| = \ln|t| + C'$. Multiplying by -2 and using rules of logs, this is $\ln|1-y^2| = \ln(|t|^{-2}) + C'$, so that, exponentiating and letting $C = \pm e^{C'}$, $1 - y^2 = Ct^{-2}$. Thus

$$y = \pm\sqrt{1 - Ct^{-2}}.$$