

3. Choose from among the given phase portraits the phase portrait for the system $\mathbf{x}' = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix} \mathbf{x}$ for each given value of a (1 point each, no justification needed for this problem):

(a) $a = 0$. Phase Portrait #

The eigenvalues in general are $\lambda = \frac{1}{2}(3 \pm \sqrt{16a+1})$. Taking $a = 0$ gives $\lambda = 1, 2$ so we have an unstable node.

(b) $a = \frac{15}{16}$. Phase Portrait #

For $a = \frac{15}{16}$ the eigenvalues are $\lambda = -\frac{1}{2}, \frac{7}{2}$, so we have a saddle point.

(c) $a = -\frac{26}{16}$. Phase Portrait #

Taking $a = -\frac{26}{16}$ the eigenvalues are $\lambda = \frac{1}{2}(3 \pm 5i)$, so we have an unstable spiral point.

