

5. (4 points) Consider the initial-value problem for $y(t)$: $y' = \cos(\pi y) + 1$, $y(3) = -2$. Does $\lim_{t \rightarrow +\infty} y(t)$ exist? If so, what is it?

Solution: the equilibria are the roots of $\cos(\pi y) + 1$ which are all of the odd integers. Since $\cos(\pi y) + 1 \geq 0$, each of these roots is a semistable equilibrium, attractive from the left and repulsive on the right. So by the phase line method and the fact that $y(3) = -2$ lies halfway between two equilibria,

$$\lim_{t \rightarrow +\infty} y(t) = -1.$$