6. [16 points] Consider a animal population modeled by a differential equation P' = f(P), where the function f(P) involves a parameter k. At k = 1 there is a bifurcation point, as shown in the bifurcation diagram to the right. In this figure, solid curves indicate stable solutions while dashed curves indicate unstable ones. Even though P < 0 is not physically realizable, include negative values of P in your analysis in parts (a) and (b) below.



**a**. [6 points] Sketch phase diagrams for the differential equation P' = f(P) for k = 0.5, k = 1 and k = 1.5.

**b**. [6 points] Sketch qualitatively reasonable solution curves this equation for the case k = 1.5.

c. [4 points] Thinking of P as an animal population, what is the implication of the bifurcation point? Give a possible explanation for what k could measure.