1. [14 points] Find explicit general, real-valued solutions for each of the following. *(Note that minimal partial credit will be given on this problem.)*

a. [7 points] $\frac{dy}{dx} = -\frac{\cos x}{\sin x}\, y + \frac{1}{\sin x}$.

Solution: This is linear and not separable, so we must use an integrating factor. The equation may be rewritten as

$$\frac{dy}{dx} + \frac{\cos x}{\sin x}\, y = \frac{1}{\sin x},$$

so the integrating factor is

$$\mu(x) = e^{\int \frac{\cos x}{\sin x}\, dx} = e^{\ln|\sin x|} = \sin x.$$

Multiplying both sides by μ, we have $(\mu \cdot y)' = 1$, so that $(\sin x)y = x + C$, and therefore

$$y = \frac{x + C}{\sin x}.$$

b. [7 points] $\frac{dy}{dx} - (x-1)y^2 = x - 1$.

Solution: This is nonlinear, but separable. Rewriting the equation, we have $y' = (x-1)(y^2+1)$, so that $\frac{y'}{y^2+1} = x - 1$. Integrating both sides, we have

$$\int \frac{y'}{y^2+1}\, dx = \int x - 1\, dx, \quad \text{or}$$

$$\arctan(y) = \frac{1}{2}x^2 - x + C, \quad \text{so}$$

$$y = \tan(\frac{1}{2}x^2 - x + C).$$