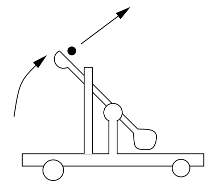

4. [10 points] Suppose we launch a 16 lb bowling ball from a catapult, as suggested in the figure to the right. In this problem we consider the vertical velocity v of the bowling ball. We shall assume that the initial vertical velocity is 45 ft/s, and that the bowling ball is released from a height of 50 ft. Gravity provides a downward acceleration of 32 ft/s^2, and the force of air resistance is proportional to the square of the velocity with constant of proportionality $k = 0.0005$. With these assumptions, the bowling ball reaches its apogee (highest point) of $h = 80.7$ ft at $t = 1.38$ seconds.

a. [6 points] Write an initial value problem for the vertical velocity of the bowling ball on its ascent. Note that you do not need to solve this problem.

Solution: In this case we have $v(0) = 45$. We can set up a differential equation by taking $ma = mv' = \sum(\text{forces})$. The forces are gravity ($= -mg$, so that it points downward) and air resistance ($= -0.0005v^2$, similarly pointing downward), so that

$$mv' = -32m - 0.0005v^2.$$

The mass is $m = 16\text{ lb}/32 = 1/2$ slugs, so that this becomes

$$v' = -32 - 0.001v^2.$$

b. [4 points] Write an initial value problem for the vertical velocity of the bowling ball on its descent. Note that you do not need to solve this problem.

Solution: In this case, the velocity will be negative and so the force of air resistance must point upwards. Our differential equation is

$$v' = -32 + 0.001v^2,$$

and we apply the initial condition $v(1.38) = 0$.