5. [16 points] Consider a differential equation $y' = f(x, y)$ with initial condition $y(0) = 1$. Using two different numerical methods, we obtain the following approximations to the solution of this initial value problem. Note that the error in the approximations is included in the tables.

Method 1:

x	0	0.5	1.0	1.5	2.0
y	1	1	1.1980	1.4238	1.5949
error	0	0.1071	0.1408	0.0794	-0.0358

Method 2:

x	0	0.5	1.0	1.5	2.0
y	1	1.1137	1.3365	1.4558	1.4854
error	0	-0.0066	0.0023	0.0475	0.0736

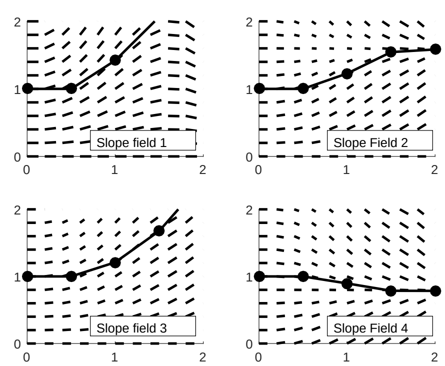
a. [3 points] What is the value of h used in the numerical approximations?

Solution: We note that the increments in x are $\Delta x = 0.5$, so $h = 0.5$.

b. [7 points] One of the methods shown is Euler's method, and the other is improved Euler. Which is which? Why?

Solution: The first method is Euler's method, and the second improved Euler. We can make a weak argument for this from the given errors: in general we expect the improved Euler method to be more accurate than Euler's method, and for most of the data points this is the case if the first method is Euler's method and the second is the improved Euler method. A strong argument comes if we note that for the first method we have $y(0.5) \approx 1 = y(0)$. For the improved Euler method to give this result, we must have $f(0, 1) = f(0.5, 1) = 0$, in which in which case Euler's method would give $y(0.5) = y(1.0) \approx 1$, which is not the case here. Thus method 1 must be Euler's method, and method 2 the improved Euler method.

c. [6 points] Given the data above, which of the slope fields to the right could be the slope field for this differential equation? Explain.



Solution: The correct slope field is slope field 2. We can see this by estimating the Euler's method solution using each slope field, as shown in the figures. For slope fields 1 and 3, the slope fields indicate that Euler's method will overshoot the limiting value shown in the table above; for slope field 4, the slopes are negative, which is not shown in the table. Slope field 2, however, matches the data well.