3. [14 points] Lake Michigan has a volume of about 4,900 km^3 of water. Each year about 158 km^3 of that flows out to Lake Huron, and we may assume that an equal amount of water flows in from the rivers feeding the lake, rainfall and snowmelt. (Of course, the loss should really take into account evaporation as well, but ignore that here.)

a. [4 points] Write a differential equation modeling the amount $p(t)$ of a pollutant in the lake, assuming that the pollutant is added at a constant rate I_0 per year.

b. [6 points] For this and part **(c)** suppose that the equation that you obtained in **(a)** is $p' + \frac{1}{20}\,p = I_0$, and that the rate at which pollutant is added changes at at $t = 4$ as regulations on allowed pollution released are loosened. Thus, instead of a constant I_0, we have $I_0(t) = \begin{cases} 100, & t < 4 \\ 1000, & t \geq 4 \end{cases}$. Find $p(t)$ if $p(0) = 500$. You need not simplify any constants in your answer.

c. [4 points] For the initial value problem you solved in **(b)**, on what domain does the solution exist, and where is it unique? On what domain would we expect a unique solution given our existence and uniqueness theorem? Is our result here consistent with the theorem?