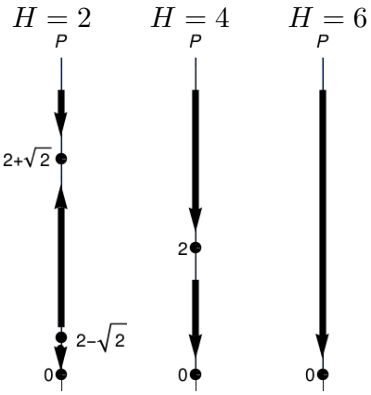
4. [14 points] Consider a population P that is modeled by the first-order differential equation $P' = f(P)$. In this problem we consider only $P \geq 0$, as a negative population is not physically relevant.

a. [4 points] If the phase line for the population is shown to the right, what could the differential equation be? Why?

b. [6 points] Now suppose that $f(P)$ depends on a parameter H, which measures the amount of harvesting of the population (e.g., if the population was fish, H could measure how many of the fish are caught through fishing). If the phase lines for $H = 2$, $H = 4$, and $H = 6$ are shown to the right, which, if any, of the following equations could model the population? Explain.
i. $P' = -P(P-1)(P-H)$ ii. $P' = P^3 - 4P^2 + HP$
iii. $P' = -P(P^2 - HP + 4)$ iv. $P' = -P(P^2 - 4P + H)$



c. [4 points] Finally, sketch a qualitatively accurate plot of solutions to the equation for the case $H = 4$.