6. [12 points] Identify each of the following as true or false, by circling "True" or "False" as appropriate, and provide a short (one or two sentence) explanation indicating why you selected that answer.

a. [3 points] The initial value problem $(y^2 - 1)y' = (t - 1)$, $y(0) = 0$, is guaranteed to have a unique solution for all times $t > 0$.

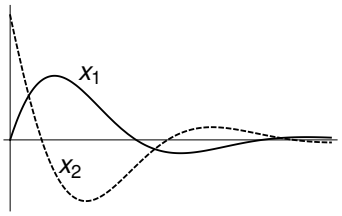
True False

b. [3 points] If the eigenvalues of a 2×2 constant, real-valued matrix $\mathbf{A}$ are $\lambda_1 = 0$ and $\lambda_2 = 1$, then the system of algebraic equations $\mathbf{Ax} = \mathbf{0}$ has infinitely many nonzero solutions.

True False

c. [3 points] If $\mathbf{A} = \begin{pmatrix} -1 & a \\ -a & -1 \end{pmatrix}$, then component plots for the system of equations $\mathbf{x}' = \mathbf{Ax}$ will appear as in the figure to the right for all real values of a.

True False



d. [3 points] A first-order problem such as $y' = t\sin(y) + \cos(y)$, which is neither linear nor separable, is amenable to qualitative analysis by drawing a phase line and sketching qualitatively accurate solution curves.

True False