7. [16 points] The van der Pol equation has the form $x'' + \mu \frac{df}{dx} x' + x = 0$. In this problem suppose that $f(x) = -\sin(x)$, so that the equation becomes $x'' - \mu \cos(x) x' + x = 0$.

a. [4 points] Letting $x_1 = x$ and $x_2 = x'$, write this as a system of two first-order differential equations in x_1 and x_2.

b. [4 points] Use a Taylor expansion to linearize the original equation at the critical point $x = 0$.

Problem 7, continued.

c. [4 points] Suppose that the equation you obtained in **(b)** is, for some value of μ,

$$x'' + 3x' + 2x = 0.$$

Write this as a matrix equation in $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and solve it.

d. [4 points] Sketch a phase portrait given your solution in **(c)**. What does it tell us about the long-term behavior of the current x in the circuit?