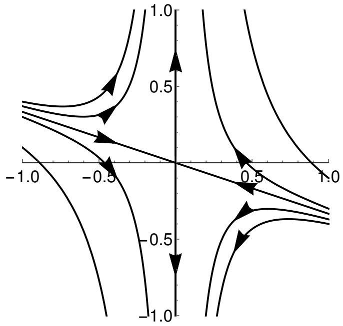
5. [15 points] For each of the following the given figure is a phase portrait for a system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is a constant 2×2 matrix. For each select the correct characterization of the eigenvalues of $\mathbf{A}$ and fill in the requested information about an eigenvector of this matrix.

a. [5 points]



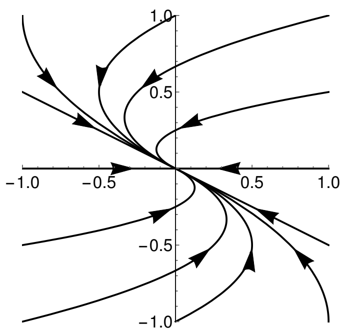
The eigenvalues of $\mathbf{A}$ could be (circle one):

$\lambda_1 = 1,\ \lambda_2 = 2;$ $\boxed{\lambda_1 = -1,\ \lambda_2 = 2;}$
$\lambda_1 = -1,\ \lambda_2 = -2;$ $\lambda_{1,2} = 1 \pm i;$
$\lambda_{1,2} = -1 \pm i$

If possible, give one eigenvector of $\mathbf{A}$ (if it is not possible, write "n/a"): $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Solution: There are trajectories approaching and leaving the origin, so we must have a positive and a negative value of λ. Trajectories leave the origin along the y-axis, so one eigenvector must be $\mathbf{v} = \begin{pmatrix} 0 & 1 \end{pmatrix}^T$. (The other has trajectories which converge to the origin, and is $\mathbf{v} = \begin{pmatrix} 3 & -1 \end{pmatrix}^T$, but this isn't possible to determine exactly.)

b. [5 points]



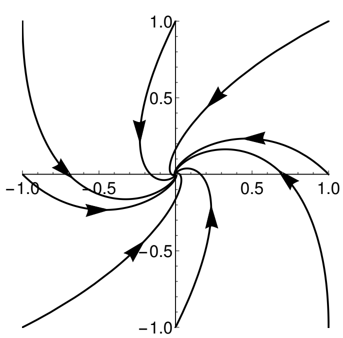
The eigenvalues of $\mathbf{A}$ could be (circle one):

$\lambda_1 = 1,\ \lambda_2 = 2;$ $\lambda_1 = -1,\ \lambda_2 = 2;$
$\boxed{\lambda_1 = -1,\ \lambda_2 = -2;}$ $\lambda_{1,2} = 1 \pm i;$
$\lambda_{1,2} = -1 \pm i$

If possible, give one eigenvector of $\mathbf{A}$ (if it is not possible, write "n/a"): $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Solution: There are two straight line trajectories (one along the x-axis), so the eigenvalues and vectors must be real, and one eigenvector must be $\mathbf{v} = \begin{pmatrix} 1 & 0 \end{pmatrix}^T$. Both eigenvalues are negative because all trajectories approach the origin. (The second eigenvector looks to be, and is, $\mathbf{v} = \begin{pmatrix} 2 & -1 \end{pmatrix}^T$.)

c. [5 points]



The eigenvalues of $\mathbf{A}$ could be (circle one):

$\lambda_1 = 1,\ \lambda_2 = 2;$ $\lambda_1 = -1,\ \lambda_2 = 2;$
$\lambda_1 = -1,\ \lambda_2 = -2;$ $\lambda_{1,2} = 1 \pm i;$
$\boxed{\lambda_{1,2} = -1 \pm i}$

If possible, give one eigenvector of $\mathbf{A}$ (if it is not possible, write "n/a"): **n/a**

Solution: There are no straight line solutions, and it appears that the trajectories all spiral in to the origin, so λ must be complex, and we cannot tell what the eigenvectors are.