5. [15 points] Consider the initial value problem $y' = -\frac{1}{2}y + \sin(y)$, $y(t_0) = y_0$.

  a. [5 points] Without trying to solve it, does this initial value problem have a solution? Does your answer depend on the values of t_0 and y_0? Explain.

  b. [5 points] By using the Taylor expansion for $\sin(y)$ near the critical point $y = 0$, write a linear equation approximating this equation and solve it. If we start with $y(0) = y_0$ with y_0 small, what does it predict will happen to the solution of the (nonlinear) problem? Is the critical point $y = 0$ stable or unstable?

  c. [5 points] Retain another term in the expansion for $\sin(y)$ and write a new differential equation that approximates the equation we started with. Find all critical points, draw a phase line, and explain what it predicts for the behavior of the system for large times.