**1**. [15 points] Find real-valued solutions to each of the following, as indicated. Where possible, find explicit solutions.

**a**. [8 points] $\ln(1/y^2)\,\frac{dy}{dt} = r\,y$, $y(0) = e$.

*Solution:* Separating variables and using rules of logarithms, we have

$$(-2\ln(y))\frac{dy}{y} = r\,dt,$$

so that on integrating both sides we have

$$-(\ln(y))^2 = r\,t + C.$$

Evaluating the initial condition, we have $-1 = C$. We therefore have the implicit solution

$$(\ln(y))^2 = 1 - r\,t.$$

Solving for $\ln(y)$, $\ln(y) = \pm\sqrt{1-rt}$, and we take the positive square root to ensure the initial condition is satisfied. Thus

$$y = e^{\sqrt{1-rt}}.$$

**b**. [7 points] Find the general solution to $t\frac{dy}{dt} = ry + t^2$.

*Solution:* Note that this is linear but not separable. Putting it in standard form, we have $\frac{dy}{dt} - \frac{r}{t}\,y = t$, so that an integrating factor is

$$\mu = e^{\int -\frac{r}{t}\,dt} = e^{-r\ln|t|} = t^{-r}.$$

Multiplying by $\mu$, we have $(y\,t^{-r})' = t^{-r+1}$, so that $y\,t^{-r} = \frac{1}{-r+2}t^{-r+2} + C$, and

$$y = \frac{1}{-r+2}\,t^2 + C\,t^r.$$