3. [14 points] In the following, the matrices $A$ and $B$ are $2 \times 2$ real-valued matrices. The vector $x$ is a $2 \times 1$ vector $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

a. [7 points] If $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and the solution to $Ax = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ is illustrated in the figure to the right, what are the eigenvalues of $A$?

**Solution:** We see that the two lines shown are $x_2 = 3 - x_1$ and $x_2 = 3x_1 - 1$. Because the right-hand side of the system of equations given is $(3 \quad -1)$, the equations must be $x_1 + x_2 = 3$ and $-3x_1 + x_2 = -1$. Therefore the rows of $A$ must be $(1 \quad 1)$ and $(-3 \quad 1)$, and because the solution shown is $(1,2)$, the matrix $A$ must be $A = \begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix}$.

The eigenvalues are determined by $(1 - \lambda)(1 - \lambda) + 3 = (\lambda - 1)^2 + 3 = 0$, so that $\lambda = 1 \pm i\sqrt{3}$.

b. [7 points] Suppose that $B \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $B \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. What is the general solution to $x' = Bx$?

**Solution:** These two systems tell us that the eigenvalues and eigenvectors of $B$ are $\lambda = 3$ and $\lambda = -2$, with $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. The general solution is therefore $x = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-2t}$. 