4. [15 points] Consider the system of differential equations given by $\mathbf{x}' = \mathbf{P}(t)\mathbf{x}$ with the initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$.

a. [4 points] If $\mathbf{P}(t) = \begin{pmatrix} 0 & 1 \\ -2t^{-2} & 2t^{-1} \end{pmatrix}$, is this a linear or nonlinear problem? If we apply the initial condition, will there be a unique solution? Explain.

b. [6 points] If $\mathbf{P}(t) = \mathbf{A}$, a 2×2 constant real-valued matrix, and if a general solution to the system is $\mathbf{x} = c_1\mathbf{v}_1 e^{\lambda t} + c_2(t\mathbf{v}_1 + \mathbf{v}_2)e^{\lambda t}$, how many solutions are there to each of the following algebraic systems of equations? Why?

(i) $\mathbf{A}\mathbf{x} = 2\lambda\mathbf{x}$

(ii) $(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = \mathbf{v}_1$

c. [5 points] If $\mathbf{P}(t) = \mathbf{B}$, a 2×2 constant real-valued matrix, and a solution to the system is $\mathbf{x} = \begin{pmatrix} \cos(3t) \\ \cos(3t) - 2\sin(3t) \end{pmatrix} e^{-4t}$, what are the eigenvalues and eigenvectors of $\mathbf{B}$?