

6. [15 points] In lab 1 we considered the Gompertz equation,  $y' = r y \ln(K/y)$ . We explore this further in this problem.

a. [5 points] Consider the initial condition  $y(0) = 1$ . Find a linear approximation to the Gompertz equation that is valid near this initial condition. Under what conditions would you expect your approximation to be accurate?

b. [5 points] We found that for  $y$  near  $K$ , the Gompertz equation is approximated as  $y' = -rK(y - K)$ . Solve this and explain what its solution tells us about solutions to the Gompertz equation.

c. [5 points] If we retain two terms from the Taylor expansion of  $\ln(K/y)$  near  $y = K$ , we obtain the cubic differential equation  $y' = f(y)$ , where  $f(y)$  is shown in the figure to the right. Sketch a phase line for this equation and explain what it suggests about the long-term behavior of the tumor.

