1. [15 points] Solve each of the following, finding explicit real-valued solutions as indicated.

a. [7 points] Find the general solution to \( y' = \frac{5 + 5s^5 - 5s^4y}{1 + s^5} \).

Solution: Simplifying the fraction on the right-hand side, this is \( y' = 5 - \frac{5s^4}{1 + s^5} y \), which is a first-order linear problem. In standard form, this is \( y' + \frac{5s^4}{1 + s^5} y = 5 \), so (noting that \( \int \frac{5s^4}{1 + s^5} \, ds = \ln |1 + s^5| \)) an integrating factor is \( \mu = 1 + s^5 \). Multiplying both sides by \( \mu \), \((\mu y)' = 5 + 5s^5 \). Integrating, \((1 + s^5) y = 5s + \frac{5}{6}s^6 + C \), so that

\[
y = \frac{5s + \frac{5}{6}s^6 + C}{1 + s^5}.
\]

b. [8 points] Solve the initial value problem \( R' = (2 - 10z)R^2 \), \( R(0) = -2 \).

Solution: This is first-order and nonlinear, but separable. Separating, we have \( R'/R^2 = 2 - 10z \), so that \(-R^{-1} = 2z - 5z^2 + C \), and

\[
R = -\frac{1}{2z - 5z^2 + C}.
\]

For \( R(0) = -2 \), \( C = \frac{1}{2} \), and

\[
R = -\frac{1}{2z - 5z^2 + \frac{1}{2}}.
\]