1. [15 points] Solve each of the following, finding explicit real-valued solutions as indicated.

a. [7 points] Find the general solution to $y' = \dfrac{5 + 5s^5 - 5s^4 y}{1 + s^5}$.

Solution: Simplifying the fraction on the right-hand side, this is $y' = 5 - \frac{5s^4}{1+s^5}\,y$, which is a first-order linear problem. In standard form, this is $y' + \frac{5s^4}{1+s^5}\,y = 5$, so (noting that $\int \frac{5s^4}{1+s^5}\,ds = \ln|1 + s^5|$) an integrating factor is $\mu = 1 + s^5$. Multiplying both sides by μ, $(\mu\, y)' = 5 + 5s^5$. Integrating, $(1 + s^5)y = 5s + \frac{5}{6}s^6 + C$, so that

$$y = \frac{5s + \frac{5}{6}s^6 + C}{1 + s^5}.$$

b. [8 points] Solve the initial value problem $R' = (2 - 10z)R^2$, $R(0) = -2$.

Solution: This is first-order and nonlinear, but separable. Separating, we have $R'/R^2 = 2 - 10z$, so that $-R^{-1} = 2z - 5z^2 + C$, and

$$R = -\frac{1}{2z - 5z^2 + C}.$$

For $R(0) = -2$, $C = \frac{1}{2}$, and

$$R = -\frac{1}{2z - 5z^2 + \frac{1}{2}}.$$