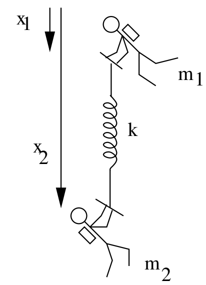

7. [15 points] The figure to the right shows two (hypothetical) skydivers, with a spring connecting them. We assume that the mass of the first, m_1, is less than the mass of the second, m_2. The distances that each has fallen are x_1 and x_2, and the spring constant is k. Let L be the equilibrium length of the spring. Then the system is modeled as

$$x_1'' = \frac{k}{m_1}(-x_1 + x_2) + \left(g - \frac{kL}{m_1}\right)$$
$$x_2'' = \frac{k}{m_2}(x_1 - x_2) + \left(g + \frac{kL}{m_2}\right).$$

a. [3 points] If we write this as a matrix equation $\mathbf{x}'' = \mathbf{A}\mathbf{x} + \mathbf{f}$, what are $\mathbf{x}$, $\mathbf{A}$ and $\mathbf{f}$?

b. [4 points] Now suppose that we're interested in finding the solution to the homogeneous problem associated with this system. If we take $\mathbf{x} = \mathbf{v}e^{\omega t}$, what equation must $\mathbf{v}$ and ω satisfy? How are $\mathbf{v}$ and ω related to the matrix $\mathbf{A}$ that you found above?

c. [8 points] Now suppose that the eigenvalues and eigenvectors of the matrix $\mathbf{A}$ you found in (a) are $\lambda_1 = 0$, with $\mathbf{v}_1 = \begin{pmatrix}1\\1\end{pmatrix}$ and $\lambda_2 = -4$ with $\mathbf{v}_2 = \begin{pmatrix}3\\1\end{pmatrix}$. Write the complementary homogeneous solution to your system.