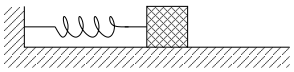
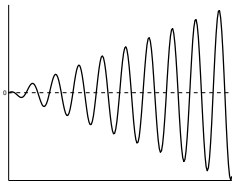
4. [12 points] Consider the differential equation $x'' + a\,x' + b\,x = A_0\cos(\omega t)$, modeling dispacement x of the mass in the mass-spring system shown to the right. In this equation, a, b, A_0 and ω are constant parameters.

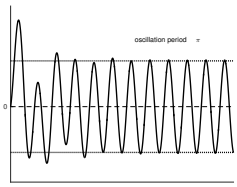


a. [6 points] If a representative graph of x as a function of time t is shown in the figure to the right, can you determine if any of a, b, A_0 or ω must be zero or must be non-zero? Must any of a, b, A_0 or ω be related in any way? Can you tell what value any of them must have?



Solution: This figure shows resonance, so we know $a = 0$, $b = \omega^2$ and $A_0 \neq 0$. We are unable to tell any specific values for these.

b. [6 points] If a representative graph of x as a function of time t is shown in the figure to the right, can you determine if any of a, b, A_0 or ω must be zero or must be non-zero? Must any of a, b, A_0 or ω be related in any way? Can you tell what value any of them must have?



Solution: This figure shows a transient motion followed by a steady-state oscillation, so we know $a, b > 0$ and $A_0 \neq 0$. The frequency ω determines the period of the steady state oscillation, so $\omega = 2$.