4. [12 points] Consider the differential equation $x'' + ax' + bx = A_0 \cos(\omega t)$, modeling displacement $x$ of the mass in the mass-spring system shown to the right. In this equation, $a$, $b$, $A_0$ and $\omega$ are constant parameters.

a. [6 points] If a representative graph of $x$ as a function of time $t$ is shown in the figure to the right, can you determine if any of $a$, $b$, $A_0$ or $\omega$ must be zero or must be non-zero? Must any of $a$, $b$, $A_0$ or $\omega$ be related in any way? Can you tell what value any of them must have?

Solution: This figure shows resonance, so we know $a = 0$, $b = \omega^2$ and $A_0 \neq 0$. We are unable to tell any specific values for these.

b. [6 points] If a representative graph of $x$ as a function of time $t$ is shown in the figure to the right, can you determine if any of $a$, $b$, $A_0$ or $\omega$ must be zero or must be non-zero? Must any of $a$, $b$, $A_0$ or $\omega$ be related in any way? Can you tell what value any of them must have?

Solution: This figure shows a transient motion followed by a steady-state oscillation, so we know $a, b > 0$ and $A_0 \neq 0$. The frequency $\omega$ determines the period of the steady state oscillation, so $\omega = 2$. 