7. [14 points] Recall the linearized version of our laser model,

\[ u' = -\gamma(Au + v) \]
\[ v' = (A - 1)u. \]

Consider the case of constant forcing \((A = \text{a constant})\) and the initial conditions \(u(0) = u_0, v(0) = 0\).

a. [6 points] Find a system of algebraic equations for \(U(s) = \mathcal{L}\{u(t)\}\) and \(V(s) = \mathcal{L}\{v(t)\}\).

b. [4 points] Solve your system from (a) to find \(U(s)\) and \(V(s)\).

\text{(If you are unable to solve (a), consider the system } (s+a)U+bV = u_0, -cU+(s+a)V = 0. \text{)}

c. [4 points] Recall that in the lab we solved the characteristic equation \(\lambda^2 + \gamma A\lambda + \gamma (A-1) = 0\), finding \(\lambda^2 + \gamma A\lambda + \gamma (A-1) = (\lambda + \frac{1}{2} \lambda A)^2 + (-\frac{1}{4} \gamma^2 A^2 + \gamma (A-1)) = (\lambda + \mu)^2 + \nu^2 = 0\) (so that \(\lambda = -\mu \pm i\nu\)). Use this to rewrite your solutions in (b) in terms of \(\mu\) and \(\nu\). Find \(u(t) = \mathcal{L}^{-1}\{U(s)\}\) and \(v(t) = \mathcal{L}^{-1}\{V(s)\}\) in terms of \(\mu\) and \(\nu\).

\text{(If you are stuck, assume the denominator of your } U \text{ and } V \text{ is of the form } (s + a)^2 + b^2. \text{)}