7. [14 points] Recall the linearized version of our laser model,

$$u' = -\gamma(Au + v)$$
$$v' = (A-1)\,u.$$

Consider the case of constant forcing (A = a constant) and the initial conditions $u(0) = u_0$, $v(0) = 0$.

a. [6 points] Find a system of algebraic equations for $U(s) = \mathcal{L}\{u(t)\}$ and $V(s) = \mathcal{L}\{v(t)\}$.

b. [4 points] Solve your system from **(a)** to find $U(s)$ and $V(s)$.
(If you are unable to solve (a), consider the system $(s+a)U+bV = u_0$, $-cU+(s+a)V = 0$.*)*

c. [4 points] Recall that in the lab we solved the characteristic equation $\lambda^2+\gamma A\lambda+\gamma(A-1) = 0$, finding $\lambda^2 + \gamma A\lambda + \gamma(A-1) = (\lambda + \frac{1}{2}\,\lambda A)^2 + (-\frac{1}{4}\gamma^2 A^2 + \gamma(A-1)) = (\lambda+\mu)^2 + \nu^2 = 0$ (so that $\lambda = -\mu \pm i\nu$). Use this to rewrite your solutions in **(b)** in terms of μ and ν. Find $u(t) = \mathcal{L}^{-1}\{U(s)\}$ and $v(t) = \mathcal{L}^{-1}\{V(s)\}$ in terms of μ and ν.
(If you are stuck, assume the denominator of your U and V is of the form $(s+a)^2 + b^2$.*)*