1. [16 points] Find real-valued solutions for each of the following, as indicated. *(Note that minimal partial credit will be given on this problem.)*

a. [8 points] Find the general solution to $y'' + 4y' + 13y = 26t$.

Solution: The general solution will be $y = y_c + y_p$, where y_c solves the complementary homogenous problem and y_p is a particular solutions. For y_c we guess $y = e^{\lambda t}$, so that $\lambda^2+4\lambda+13 = (\lambda+2)^2+9 = 0$, and $\lambda = -2\pm 3i$. Thus $y_c = c_1e^{-2t}\cos(3t)+c_2e^{-2t}\sin(3t)$. For y_p we use the method of undetermined coefficients, taking $y_p = At + B$. Plugging in, we have

$$0 + 8A + 13At + 13B = 26t,$$

so that $A = 2$ and $B = -8/13$. Thus the general solution is

$$y = c_1e^{-2t}\cos(3t) + c_2e^{-2t}\sin(3t) + 2t - \frac{8}{13}.$$

b. [8 points] Solve $y'' + 6y' + 8y = e^{-2t}$, $y(0) = 0$, $y'(0) = 0$.

Solution: Again, with $y = e^{\lambda t}$, we have $\lambda^2 + 6\lambda + 8 = (\lambda + 2)(\lambda + 4) = 0$, so that $y_c = c_1e^{-2t} + c_2e^{-4t}$. Our guess for y_p is then $y_p = Ate^{-2t}$ (where we have to multiply by t because the forcing appears in the complementary homogeneous solution). The derivatives of y_p are $y_p' = Ae^{-2t} - 2Ate^{-2t}$ and $y_p'' = -4Ae^{-2t} + 4Ate^{-2t}$. Plugging in, we have

$$(-4Ae^{-2t} + 4Ate^{-2t}) + (6Ae^{-2t} - 12Ate^{-2t}) + 8Ate^{-2t} = 2Ae^{-2t} = e^{-2t},$$

so that $A = \frac{1}{2}$. Thus the general solution is

$$y = c_1e^{-2t} + c_2e^{-4t} + \frac{1}{2}\,t\,e^{-2t}.$$

Plugging in the initial conditions, $y(0) = c_1 + c_2 = 0$, and $y'(0) = -2c_1 - 4c_2 + \frac{1}{2} = 0$. Thus $c_1 = -c_2$, so that $c_2 = \frac{1}{4}$ and $c_1 = -\frac{1}{4}$, and $y = -\frac{1}{4}\,e^{-2t} + \frac{1}{4}\,e^{-4t} + \frac{1}{2}\,t\,e^{-2t}$.