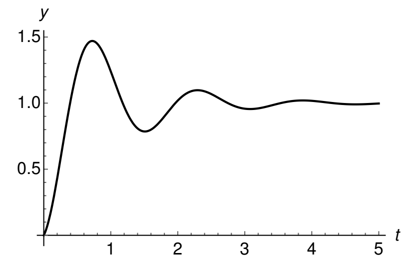
5. [14 points] For the first two of the following, identify each as true or false, by circling "True" or "False" as appropriate, and provide a short (one sentence) explanation indicating why you selected that answer. For the last give a short answer explaining the indicated question.

a. [4 points] For some constant ω and k, a solution to the mechanical system $y'' + 2y' + ky = \cos(\omega t)$ could be that shown to the right.



Solution: This cannot be true; the forcing requires that the steady state solution be sinusoidal, and centered on the t-axis.

True [False]

b. [4 points] Let $F(s) = \frac{s^2+1}{s^2+3s+5}$. There is some piecewise continuous function $f(t)$, of exponential order, for which $\mathcal{L}\{f(t)\} = F(s)$.

True [False]

Solution: This is false, because $F(s) \to 1 \neq 0$ as $s \to \infty$. We know that all transforms of regular functions must go to zero as $s \to \infty$.

c. [6 points] Your friends Anna and Andrew are solving the two problems $y'' + 0.1y' + y = 0$, $y(0) = 0$, $y'(0) = 1$ and $z'' + 0.1z' + z = \delta(t-3)$, $z(0) = 0$, $z'(0) = 0$. Anna thinks that $z(t) = y(t-3)$, while Andrew thinks they are different. Explain why they are both partly correct.

Solution: Note that the transforms of these problems give $Y = 1/(s^2 + 0.1s + 1)$ and $Z = e^{-3s}/(s^2 + 0.1s + 1)$. Thus we know that $z(t) = y(t-3)u_3(t)$. The two are the same, with the ambiguity of the value of the derivative at $t = 3$—because z has the step function there the value of z' at $t = 3$ is not uniquely determined.