- **2.** [12 points] The following problems consider a non-homogeneous second-order linear differential equation L[y] = g(t). Suppose that  $y_1$  and  $y_2$  are solutions to this equation, and that  $y_3$ ,  $y_4$ , and  $y_5$  are solutions to the complementary homogeneous problem L[y] = 0.
  - **a**. [3 points] Can you say what problem each of the following solve? If so, indicate what it is; if not, write "none." (No explanation necessary.)

i.  $y_1 - y_2$ 

ii.  $y_1 - y_3$ 

- iii.  $y_1 + y_2 + y_3 + y_4 + y_5$
- **b**. [3 points] Explain how you are able to determine your answers in (a), or why it is not possible to tell.

- c. [6 points] The following statements are not guaranteed to be true. Explain why.
  - i. The solution to the initial problem L[y] = 0,  $y(0) = y_0$ ,  $y'(0) = v_0$  (for any  $y_0$  and  $v_0$ ) can be written as  $y = c_1y_3 + c_2y_4 + c_3y_5$  for some  $c_1$ ,  $c_2$ , and/or  $c_3$  (where one or more of  $c_1$ ,  $c_2$ , and  $c_3$  may be zero).
  - ii. Because both  $y_1$  and  $y_2$  satisfy L[y] = g(t), we must have  $y_1 = y_2$ .