2. [12 points] The following problems consider a non-homogeneous second-order linear differential equation $L[y]=g(t)$. Suppose that $y_{1}$ and $y_{2}$ are solutions to this equation, and that $y_{3}, y_{4}$, and $y_{5}$ are solutions to the complementary homogeneous problem $L[y]=0$.
a. [3 points] Can you say what problem each of the following solve? If so, indicate what it is; if not, write "none." (No explanation necessary.)
i. $y_{1}-y_{2}$
ii. $y_{1}-y_{3}$
iii. $y_{1}+y_{2}+y_{3}+y_{4}+y_{5}$
b. [3 points] Explain how you are able to determine your answers in (a), or why it is not possible to tell.
c. [6 points] The following statements are not guaranteed to be true. Explain why.
i. The solution to the initial problem $L[y]=0, y(0)=y_{0}, y^{\prime}(0)=v_{0}$ (for any $y_{0}$ and $v_{0}$ ) can be written as $y=c_{1} y_{3}+c_{2} y_{4}+c_{3} y_{5}$ for some $c_{1}, c_{2}$, and/or $c_{3}$ (where one or more of $c_{1}, c_{2}$, and $c_{3}$ may be zero).
ii. Because both $y_{1}$ and $y_{2}$ satisfy $L[y]=g(t)$, we must have $y_{1}=y_{2}$.
