4. [14 points] Recall that the nonlinear model for the number of photons P and population inversion N in a ruby laser that we considered in lab 3 had an equilibrium point $(P, N) = (1, A-1)$. If we assume that $A = A_0 + a\cos(\omega t)$, with a a very small value, the dynamics of the system near the equilibrium point are modeled by the linear system $u' = -\gamma(Au + v) + \gamma a\cos(\omega t)$, $v' = (A-1)u$

a. [4 points] Rewrite this linear system as a second order equation in v.

b. [6 points] Suppose that the second order equation you obtain in (a) is $v'' + 2v' + v = \cos(\omega t)$, so that the solution to the complementary homogeneous problem is $v_c = c_1 e^{-t} + c_2 t e^{-t}$. Set up the solution for v_p using variation of parameters, and solve them to obtain explicit equations for u_1' and u_2' in terms of t only. *(Do not solve these to find u_1 and u_2.)*

c. [4 points] It turns out that, for some A and B, $v_p = A\cos(\omega t) + B\sin(\omega t)$. Representative values of A and B are given for different values of ω in the table below. Does the system exhibit resonance? Write the response v_p to a forcing of $\cos(2t)$ in phase-amplitude form.

$\omega =$	1	2	3	4	5
$A =$	0	-0.12	-0.08	-0.06	-0.04
$B =$	1	0.64	0.36	0.22	0.15