6. [15 points] Complete each of the following problems having to do with the Laplace transform.

  a. [5 points] Find the inverse Laplace transform of $F(s) = \dfrac{5s}{s^2 + 4s + 6}$

  b. [5 points] Given that $F(s) = \mathcal{L}\{f(t)\}$, use the integral definition of the Laplace transform to derive the transform rule $-F'(s) = \mathcal{L}\{tf(t)\}$.

  c. [5 points] Consider the initial value problem $ty'' + y = 0$, $y(0) = 1$, $y'(0) = 0$. If $Y = \mathcal{L}\{y\}$, what equation does Y satisfy?