1. [15 points] For each of the following, find explicit real-valued solutions as indicated.
   a. [7 points] Find the solution to the initial value problem \( y'' + 2y' + 17y = 0, \ y(0) = 1, \ y'(0) = 0. \)
      
      Solution: The solution will look like \( y = e^{\lambda t}. \) Plugging in, we have the characteristic equation \( \lambda^2 + 2\lambda + 17 = (\lambda + 1)^2 + 16 = 0. \) Thus \( \lambda = -1 \pm 4i. \) A general solution is \( y = c_1 e^{-t} \cos(4t) + c_2 e^{-t} \sin(4t). \) Applying the initial conditions, we have \( c_1 = 1, \) and \( -1 + 4c_2 = 0, \) so that \( c_2 = \frac{1}{4}. \) Thus the solution is 
      
      \[ y = e^{-t} \cos(4t) + \frac{1}{4} e^{-t} \sin(4t). \]
      
      We can, of course, also solve this with Laplace transforms. The forward transform gives, with \( Y = \mathcal{L}\{y\}, \) \( s^2 Y - s + 2sY + 17Y = 0, \) so that \( Y = \frac{s(s+1)-1}{(s+1)^2+16} = \frac{s}{(s+1)^2+16} - \frac{1}{(s+1)^2+16}. \) The two terms invert using the rules for cosine, sine, and \( F(s-c) \) to give the result above.

   b. [8 points] Find the general solution to the equation \( y'' + 5y' + 4y = e^t + 8t. \)
      
      Solution: The complementary homogeneous solution will be a linear combination of exponentials \( y = e^{\lambda t}. \) Plugging in, we have \( \lambda^2 + 5\lambda + 4 = (\lambda + 4)(\lambda + 1) = 0, \) so \( \lambda = -4 \) or \( \lambda = -1 \) and \( yc = c_1 e^{-4t} + c_2 e^{-t}. \)
      
      To find the particular solution, consider the exponential and linear term separately. For the former, we guess \( yp_1 = ae^t, \) so that \( (1 + 5 + 4)ae^t = 10ae^t, \) and \( a = \frac{1}{10}. \) For the latter, we guess \( yp_2 = a_0 + a_1 t, \) so that \( 5a_1 + 4a_0 + 4a_1 t = 8t, \) and \( a_1 = 2, \) \( a_0 = -\frac{5}{2}. \) Thus the general solution is
      
      \[ y = c_1 e^{-4t} + c_2 e^{-t} + \frac{1}{10} e^t - \frac{5}{2} + 2t. \]