2. [15 points] In each of the following $\mathcal{L}$ is the Laplace transform operator, and, in (b), L is a linear, constant-coefficient differential operator.

a. [5 points] If $x' = 3x + 4y$ and $y' = 2x - y$, with initial conditions $x(0) = 0$ and $y(0) = 2$, find $X = \mathcal{L}\{x\}$ and $Y = \mathcal{L}\{y\}$.

b. [5 points] Suppose that when solving an equation $L[y] = f(t)$, $y(0) = y_0$, $y'(0) = v_0$ using the Laplace transform, we find

$$\mathcal{L}\{y(t)\} = Y(s) = \frac{5}{(s+1)(s+2)} + \frac{s}{(s+1)(s+2)(s^2+4)}.$$

What are L, $f(t)$, and the initial conditions y_0 and v_0?

c. [5 points] Derive the transform rule $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$ for a continuous function $f(t)$.