1. [12 points] Find each of the following.

a. [7 points] Use the integral definition of the Laplace transform to find \( F(s) = \mathcal{L}\{f(t)\} \), where

\[
f(t) = \begin{cases} 
e^{-1}, & 0 < t \leq 1 \\
e^{-t}, & 1 < t < \infty. \end{cases}
\]

**Solution:** Applying the transform, we have

\[
F(s) = \int_0^\infty f(t)e^{-st} \, dt = \int_0^1 e^{-1}e^{-st} \, dt + \int_1^\infty e^{-(s+1)t} \, dt = \frac{1}{s} e^{-1}e^{-st} \bigg|_{t=0}^{t=1} - \frac{1}{s+1} e^{-(s+1)t} \bigg|_{t=1}^{t=\infty} = \frac{1}{s}(e^{-1} - e^{-(s+1)}) + \frac{1}{s+1} e^{-(s+1)}.
\]

b. [5 points] Give another function \( g(t) \) for which \( \mathcal{L}\{g(t)\} = F(s) \). Explain your answer briefly.

**Solution:** Because the transform is defined as an integral, any single point differences between \( g \) and \( f \) will not change the transform. Thus, we could take

\[
g(t) = \begin{cases} 
e^{-1}, & 0 < t < 1 \\
1, & t = 1 \\
e^{-t}, & 1 < t < \infty, \end{cases}
\]

or any other variation where we make \( g(t) \) piecewise continuous with isolated points that have a different value from \( f(t) \).