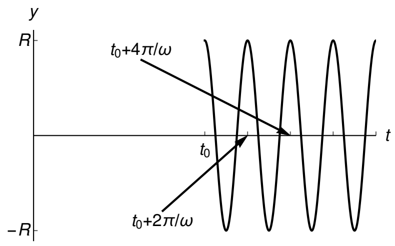
5. [12 points] In lab 4 we consider a forced electrical system of the form

$$y'' + 2\gamma y' + \omega_0^2 y = F(t),$$

which models the current in a circuit. In this problem we take $\gamma = 1$ and $\omega_0 = 3$.

a. [7 points] Carefully sketch a qualitatively accurate graph of the steady state response current to a forcing voltage $F = k\sin(\omega t)$. Explain what functions appear in the response and therefore why your graph has the form it does. As possible, give information about the relative position of significant features of your graph. *(Note that you do not need to, and probably do not want to, solve for the steady state response.)*

Solution: We know that the solution to the problem will be $y = y_c + y_p$, and because of the damping term $2\gamma y'$, the homogeneous solution will decay. Thus the steady state response will be y_p, which from the method of undetermined coefficients will be $y_p = A\cos(\omega t) + B\sin(\omega t) = R\cos(\omega t - \phi)$. Thus the steady state response is a pure sinusoidal solution with period $T = \frac{2\pi}{\omega}$, and the amplitude $R = \sqrt{A^2 + B^2}$ will depend on ω as well. Thus we have a graph such as that below. Before some starting value t_0, we will have the sum of this steady solution and the (decaying) homogenous solution.



b. [5 points] Now suppose that

$$F(t) = I(t) = \begin{cases} \frac{1}{a}, & c \le t < c + a \\ 0, & \text{otherwise,} \end{cases}$$

and that $y(0) = y'(0) = 0$. Make two sketches showing the behavior of the solution for $t > c$, first if a is large and second if a is small. In either case you will want to say something about what functions contribute to the behavior you are graphing, but need not, and probably should not, completely solve the problem.

Solution: In either case, for $c < t < c + a$ we have $y_p = \frac{1}{a\omega_0^2}$, overlayed with the homogeneous term, which will be a decaying sinusoid. For $t > c + a$, the response will decay to zero, in an oscillatory manner. If we let $a \to 0$, we know from lab that this will result in a decaying sinusoid that is equivalent to the solution to the problem with zero forcing and initial condition $y(0) = 0$, $y'(0) = 1$. Thus we have the two graphs shown below.