

4. [15 points] Note in this problem that \mathcal{L} indicates the Laplace transform.
- a. [5 points] If $f(t) = te^{-t}$, use the integral definition of the Laplace transform to find $F(s) = \mathcal{L}\{f(t)\}$.
- b. [5 points] Use rules from the table of transforms to confirm your result in (a). Be sure that it is clear what rules you are using and how they give the result you obtain.
- c. [5 points] The solution to $y'' + 3y' + 2y = e^{-t}$ with initial conditions $y(0) = 0$, $y'(0) = 2$ is $y = e^{-t} - e^{-2t} + te^{-t}$. Transform the solution to the equation and the equation itself, and show that the two expressions you get for $Y(s) = \mathcal{L}\{y(t)\}$ are the same.