5. [14 points] In lab 3 we considered the nonlinear system

$$N' = \gamma(A - N(1+P)), \qquad P' = P(N-1).$$

We established that the equilibrium solutions to the system are $(N, P) = (A, 0)$ and $(N, P) = (1, A-1)$, and that near the latter the system is approximated by the linear second order problem $v'' + \gamma v' + \gamma A(A-1)v = 0$, where v is the small variation in P from the equilibrium $A - 1$.

a. [4 points] Write the linear, second-order problem from above as a system of two linear, first-order equations.

b. [6 points] Suppose that we pick A and γ so that the characteristic equation of the linear second-order equation has a repeated root. Find the solution to the linear second-order equation in this case, and use your solution to write the solution to the system you found in (a). *(If you are stuck, assume that $A = 2$ and find a nonzero γ to finish the problem with a one point penalty.)*

c. [4 points] In Part B of the lab, we assumed that A was a function of time, that is, $A = A(t) = A_0 + 2a\cos(\omega t)$. Suppose instead we picked $A(t) = A_0 \tan(\omega t)$, so that $v'' + \gamma v' + q(t)v = 0$, with $q(t) = \gamma A(t)(A(t) - 1)$. If we start with $v(0) = 0.5$, $v'(0) = 0$, what is the longest interval on which the solution to the initial value problem is certain to have a unique solution, and why? (Note that you cannot solve the equation by hand.)

