

6. [15 points] Consider a physical system modeled by the differential equation

$$x'' + \gamma x' + kx = f(t),$$

where $x(t)$ is the physical quantity being measured and γ and k are constants.

- a. [4 points] If the physical system is underdamped, what can you say about the parameters γ and k ?

- b. [5 points] If $x(0) = x_0$, $x'(0) = v_0$, and $\mathcal{L}\{f(t)\} = F(s)$, find the transform $X(s) = \mathcal{L}\{x(t)\}$.

- c. [6 points] If $f(t) = 0$, assuming as in (a) that the system is underdamped, invert your transform from (b) to find $x(t)$. (If you are stuck, assume the equation is $x'' + \gamma x' + \gamma^2 x = 0$.)