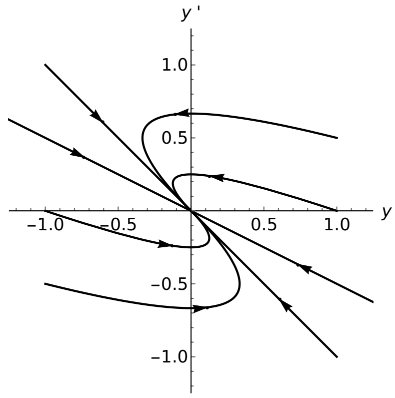
7. [12 points] In the following we consider two linear, homogeneous, second-order, constant coefficient differential equations, for $y(t)$ and $z(t)$. The phase portrait for the equation for $y(t)$ is shown to the right, and graphs of $z(t)$ for two different initial conditions are shown in the figure to the right, below. Explain in a sentence or two why each of the following **cannot** be true.



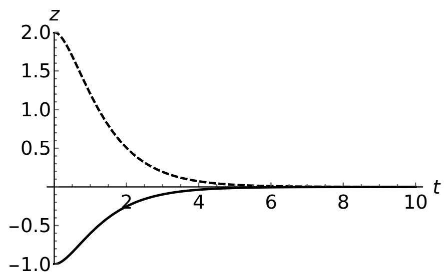
a. [3 points] The equation is $y'' - 3y' + 2y = 0$

Solution: In this case the characteristic equation is $\lambda^2 - 3\lambda + 2 = (\lambda - 2)(\lambda - 1) = 0$, so that $\lambda = 1$ or $\lambda = 2$, and solutions must grow away from the origin.

b. [3 points] The general solution to the equation is $y = c_1 e^{-t} + c_2 e^{-2t}$.

Solution: Note that if we rewrite the equation as a system, the solution is $\begin{pmatrix} y \\ y' \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-2t}$, so that the straight line solutions have to be $y = -x$ and $y = -2x$, which is not what we see here (and the direction of fastest collapse is similarly wrong).

c. [3 points] Given some initial conditions, the Laplace transform $Z(s) = \mathcal{L}\{z(t)\} = \frac{2s+4}{s^2+2s+5}$.



Solution: We see from the form of the Laplace transform that the characteristic polynomial is $p(s) = s^2+2s+5 = (s+1)^2+4$, so that roots are $s = -1 \pm 2i$, and solutions should be oscillatory, not exponentially decaying to zero.

d. [3 points] Written as a system, the equation for $z(t)$ is $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

Solution: If this were the case we have $\lambda = \pm 2i$, so the general solution to the problem is $z = c_1 \cos(2t) + c_2 \sin(2t)$, which is a pure sinusoid. This is clearly not shown here.