

3. In a lab there are five identical “RLC” circuits in each of which the capacitor charge satisfies $LQ'' + RQ' + C^{-1}Q = E(t)$ with $L = 3$ Henries, $R = 4$ Ohms, $C = 0.5$ Farads. The five circuits are driven by five time-dependent voltage sources $E(t)$ and have different amounts $Q(0)$ of initial charge on the capacitor and different amounts $Q'(0)$ of current flowing at time zero, according to this table:

Circuit	Capacitor charge	Voltage source $E(t)$	$Q(0)$	$Q'(0)$
1	$Q = Q_1(t)$	$E_1(t) = 1/(1+t^2)$ Volts	0.25 Coul.	0 Amp.
2	$Q = Q_2(t)$	$E_2(t) = 1/(1+t^2)$ Volts	0 Coul.	1 Amp.
3	$Q = Q_3(t)$	$E_3(t) \equiv 6$ Volts	0.25 Coul.	0 Amp.
4	$Q = Q_4(t)$	$E_4(t) = (25 + 24t^2)/(1+t^2)$ Volts	1 Coul.	1 Amp.
5	$Q = Q_5(t)$	For $E_5(t)$, see part (d)	0 Coul.	0 Amp.

- (a) (3 Points.) Express $Q_4(t)$ in terms of $Q_1(t)$, $Q_2(t)$, and/or $Q_3(t)$.

- (b) (2 Points.) Find $\lim_{t \rightarrow +\infty} [Q_1(t) - Q_2(t)]$.

- (c) (2 Points.) Would the answer to part (b) be different if $R = 0$ Ohms instead? Why or why not?

- (d) (3 Points.) Circuit number 5 has a “pulsed” voltage source $E(t)$ that is zero except on the time interval $1 < t < 6$ seconds, at the beginning of which it is suddenly switched on to 10 Volts, and during which it increases exponentially following $E(t) = 10e^{b(t-1)}$ for some rate $b \text{ sec}^{-1}$. If instantaneously after switching on, $E'(1) = 30 \text{ Volts/sec}$, use the definition of the Laplace transform to find $\mathcal{L}\{E(t)\}$ (but don’t solve for $Q_5(t)$).