7. (5 Points.) In solving an initial-value problem for a certain linear second-order ODE by Laplace transforms, we arrive at
$$Y(s) = \frac{s}{s^3 - 3s^2 + 9s - 27} = \frac{s}{(s-3)(s^2+9)}.$$

What is $y(t)$?

Partial fraction decomposition:
$$\frac{s}{(s-3)(s^2+9)} = \frac{A}{s-3} + \frac{Bs+C}{s^2+9}$$

$$s = A(s^2+9) + (Bs+C)(s-3)$$

At $s=3$: $3 = 18A \Rightarrow A = \tfrac{1}{6}$.

Matching s^2: $A+B=0 \Rightarrow B = -\tfrac{1}{6}$.

Matching s^1: $-3B + C = 1 \Rightarrow \tfrac{1}{2} + C = 1 \Rightarrow C = \tfrac{1}{2}$.

So
$$Y(s) = \frac{1/6}{s-3} + \frac{-\tfrac{1}{6}s + \tfrac{1}{2}}{s^2+9} = \frac{1}{6}\cdot\frac{1}{s-3} - \frac{1}{6}\cdot\frac{s}{s^2+9} + \frac{1}{6}\cdot\frac{3}{s^2+9}.$$

Taking the inverse Laplace transform:
$$y(t) = \frac{1}{6}e^{3t} - \frac{1}{6}\cos 3t + \frac{1}{6}\sin 3t = \frac{1}{6}\left(e^{3t} - \cos 3t + \sin 3t\right).$$