1. Consider an "RLC" circuit in which the capacitor charge $Q(t)$ satisfies $LQ'' + RQ' + C^{-1}Q = E(t)$ where L, R, C are the inductance (Henries), resistance (Ohms), and capacitance (Farads), and where $E(t)$ is a variable source of voltage (Volts). Suppose that $R = 2$ Ohms and $C = 1/5$ Farads, and that the voltage source is sinusoidal: $E(t) = \cos(5t)$.

   (a) (3 Points.) Find the steady-state periodic response, keeping the inductance L as a variable parameter in your answer.

   Solution: to find the steady-state periodic response we use the method of undetermined coefficients: $Q(t) = A\cos(5t) + B\sin(5t)$. Then $Q'(t) = -5A\sin(5t) + 5B\cos(5t)$ and $Q''(t) = -25A\cos(5t) - 25B\cos(5t)$, so substitution into $LQ'' + 2Q' + 5Q = \cos(5t)$ gives

$$\begin{aligned} (5-25L)A + 10B &= 1 \\ -10A + (5-25L)B &= 0 \end{aligned}$$

   where the first line comes from the terms proportional to $\cos(5t)$ and the second line comes from the terms proportional to $\sin(5t)$. Solving for A and B gives

$$A = \frac{5-25L}{(5-25L)^2+100} \quad \text{and} \quad B = \frac{10}{(5-25L)^2+100}.$$

   The steady-state periodic response is therefore

$$\boxed{Q(t) = \frac{5-25L}{(5-25L)^2+100}\cos(5t) + \frac{10}{(5-25L)^2+100}\sin(5t).}$$

   (b) (2 Points.) Find the amplitude of the steady-state periodic response from part (a), and then determine the value of inductance L that maximizes it.

   Solution: the steady-state response amplitude is $R = \sqrt{A^2+B^2}$ or

$$R = \sqrt{\frac{(5-25L)^2+100}{[(5-25L)^2+100]^2}} = \frac{1}{\sqrt{(5-25L)^2+100}}.$$

   Maximizing this is the same as minimizing the quantity under the radical, which occurs when $5 - 25L = 0$ or

$$\boxed{L = \tfrac{1}{5} = 0.2 \text{ Henries.}}$$