2. (5 Points.) For certain initial conditions, the displacement $x(t)$ of a mass from equilibrium in a mechanical system without any damping or forcing is given by

$$x(t) = -\sqrt{3}\cos(4\pi t) + \sin(4\pi t).$$

Write $x(t)$ in phase/amplitude form and use your answer to find the *second* positive time $t > 0$ at which the mass passes equilibrium. Note that for some angles in the first quadrant we have

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Solution: the amplitude is

$$R = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2,$$

and the phase satisfies $\cos(\delta) = -\sqrt{3}/2$ and $\sin(\delta) = 1/2$. So δ is an angle in the second quadrant. Since $\arccos(\cdot)$ returns an angle in the first or second quadrant, we can take $\delta = \arccos(-\sqrt{3}/2)$. Equivalently, we can relate δ to an angle θ in the first quadrant by $\delta + \theta = \pi$, for which $\cos(\theta) = \cos(\pi - \delta) = -\cos(\delta) = \sqrt{3}/2$ and $\sin(\theta) = \sin(\pi - \delta) = \sin(\delta) = 1/2$, so using the given function values, $\theta = \pi/6$. Hence $\delta = \pi - \pi/6 = 5\pi/6$. So, the phase/amplitude form is

$$\boxed{x(t) = 2\cos(4\pi t - 5\pi/6).}$$

We have $x(t) = 0$ whenever $4\pi t - 5\pi/6$ is an odd multiple of $\pi/2$, i.e., whenever $8t - 5/3$ is an odd integer, i.e., whenever $8t = 5/3$ plus an odd integer. The first positive t corresponds to taking -1 for the odd integer, and the second positive t corresponds to taking 1 for the odd integer. So, the time we seek satisfies $8t = 5/3 + 3/3 = 8/3$ or

$$\boxed{t = \tfrac{1}{3}.}$$