2. (5 Points.) For certain initial conditions, the displacement \( x(t) \) of a mass from equilibrium in a mechanical system without any damping or forcing is given by

\[
x(t) = -\sqrt{3}\cos(4\pi t) + \sin(4\pi t).
\]

Write \( x(t) \) in phase/amplitude form and use your answer to find the second positive time \( t > 0 \) at which the mass passes equilibrium. Note that for some angles in the first quadrant we have

\[
\begin{array}{c|cccc}
\theta & 0 & \frac{\pi}{6} & \frac{\pi}{4} & \frac{\pi}{3} \\
\cos(\theta) & 1 & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} & 0 \\
\sin(\theta) & 0 & \frac{1}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2}
\end{array}
\]

Solution: the amplitude is

\[
R = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2,
\]

and the phase satisfies \( \cos(\delta) = -\sqrt{3}/2 \) and \( \sin(\delta) = 1/2 \). So \( \delta \) is an angle in the second quadrant. Since \( \arccos(\cdot) \) returns an angle in the first or second quadrant, we can take \( \delta = \arccos(-\sqrt{3}/2) \).

Equivalently, we can relate \( \delta \) to an angle \( \theta \) in the first quadrant by \( \delta + \theta = \pi \), for which \( \cos(\theta) = \cos(\pi - \delta) = -\cos(\delta) = \sqrt{3}/2 \) and \( \sin(\theta) = \sin(\pi - \delta) = \sin(\delta) = 1/2 \), so using the given function values, \( \theta = \pi/6 \). Hence \( \delta = \pi - \pi/6 = 5\pi/6 \). So, the phase/amplitude form is

\[
x(t) = 2\cos(4\pi t - 5\pi/6).
\]

We have \( x(t) = 0 \) whenever \( 4\pi t - 5\pi/6 \) is an odd multiple of \( \pi/2 \), i.e., whenever \( 8t - 5/3 \) is an odd integer, i.e., whenever \( 8t = 5/3 \) plus an odd integer. The first positive \( t \) corresponds to taking \(-1\) for the odd integer, and the second positive \( t \) corresponds to taking \( 1 \) for the odd integer. So, the time we seek satisfies \( 8t = 5/3 + 3/3 = 8/3 \) or

\[
t = \frac{1}{3}.
\]