7. (5 Points.) In solving an initial-value problem for a certain linear second-order ODE by Laplace transforms, we arrive at

$$Y(s) = \frac{s}{s^3 - 3s^2 + 9s - 27} = \frac{s}{(s-3)(s^2+9)}.$$

What is $y(t)$?

Solution: We set up the partial-fraction expansion

$$Y(s) = \frac{s}{s^3 - 3s^2 + 9s - 27} = \frac{A}{s-3} + \frac{Bs + C}{s^2+9}.$$

Multiplying through by the common denominator gives $s = A(s^2+9) + (Bs+C)(s-3)$. Setting $s = 3$ gives $A = \frac{1}{6}$. Then setting $s = 0$ gives $0 = \frac{9}{6} - 3C$ so $C = \frac{1}{2}$. Matching the coefficients of the terms proportional to s gives $1 = -3B + C = -3B + \frac{1}{2}$ so $B = -\frac{1}{6}$. Therefore,

$$Y(s) = \frac{1}{6}\frac{1}{s-3} - \frac{1}{6}\frac{s}{s^2+9} + \frac{1}{2}\frac{1}{s^2+9}.$$

Multiplying and dividing the last term by 3 then allows us to use the table of transforms to find

$$y(t) = \frac{1}{6}e^{3t} - \frac{1}{6}\cos(3t) + \frac{1}{6}\sin(3t).$$