8. True or false? Write out the full word "true" or "false" and provide a brief justification (2 points each).

   (a) Variation of parameters is applicable to find a solution of $y'' + \sin(y) = \tan(t)$.
   False. This is a nonlinear problem.

   (b) The function $f(t) = e^{\sqrt{t}}$ has a Laplace transform.
   True. $e^{\sqrt{t}}$ is continuous for $t \geq 0$, and the inequality $\sqrt{t} \leq t$ holds for all $t \geq 1$, so also $e^{\sqrt{t}} \leq e^t$ holds for all $t \geq 1$. Therefore the transform exists at least for $s > 1$ (actually, for $s > 0$).

   (c) There is a piecewise-continuous function of exponential order having Laplace transform $F(s) = \dfrac{s-1}{s+1}$ for $s > -1$.
   False. The Laplace transform of every piecewise-continuous function of exponential order tends to zero as $s \to +\infty$, but $F(s) \to 1$ as $s \to +\infty$ instead. Another way to see this is to write $F(s)$ in the form $F(s) = 1 - 2/(s+1)$. Then from the table, $f(t) = \delta(t) - 2e^{-t}$, but $\delta(t)$ is not a piecewise-continuous function (it is not even a function, just a generalized function).

   (d) An unforced mechanical system described by the ODE $2y'' + y' + 2y = 0$ is underdamped.
   True. The discriminant of the characteristic equation $2\lambda^2 + \lambda + 2 = 0$ is $1^2 - 4 \cdot 2 \cdot 2 = -15 < 0$. Therefore we have distinct complex-conjugate roots and by definition the system is underdamped.

   (e) A sinusoidally-forced mechanical system described by the ODE $2y'' + y' + 2y = F_0 \cos(\Omega t)$ has a steady state sinusoidal solution with an amplitude that can be an arbitrarily large multiple of $|F_0|$ if Ω is chosen appropriately.
   False. This system has positive damping $\gamma = 1$, so the gain has a finite maximum value.