

2. [14 points] Find each of the following, providing an explicit formula where appropriate. (Note that minimal partial credit will be given on this problem.)

a. [5 points] $Y(s) = \mathcal{L}\{y(t)\}$ if $y'' + 4y' + 20y = 3\sin(2t)$, $y(0) = 1$, $y'(0) = 2$.

b. [5 points] $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

c. [4 points] Using the integral definition of the Laplace transform, derive the transform rule $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-sc}F(s)$ for a function $f(t)$ with transform $L\{f(t)\} = F(s)$. (Recall $u_c(t)$ is the unit step function at $t = c$, $u_c(t) = \begin{cases} 0, & 0 < t < c \\ 1, & t \geq c \end{cases}$.)