5. [15 points] For each of the following, identify the statement as true or false by circling “True” or “False” as appropriate, and provide a short (one or two sentence) explanation indicating why that answer is correct.

a. [3 points] For the system \( x' = -xy + y^2 \), \( y' = x^2 - xy \), the nonlinear trajectory in the phase plane with \( x(0) = -3 \) and \( y(0) = 0 \) lies on a circle centered on the origin.
   
   True
   False

b. [3 points] For a linear differential operator \( L = \frac{d^2}{dt^2} + p(t) \frac{d}{dt} + q(t) \), if \( y_1 \) and \( y_2 \) are different functions satisfying \( L[y_1] = L[y_2] = g(t) \neq 0 \), then, for any constants \( c_1 \) and \( c_2 \), \( y = c_1 y_1 - c_2 y_2 \) satisfies \( L[y] = 0 \).
   
   True
   False

c. [3 points] The solution to a differential equation \( my'' + ky = F(t) \) modeling the motion \( y \) of an undamped mechanical spring system with a periodic external force \( F(t) = F_0 \cos(\omega t) \) can always be written as \( y = A \cos(\omega_0 t - \delta_1) + B \cos(\omega t - \delta_2) \), a sum of two oscillatory terms. (\( A \), \( B \), \( \omega_0 \), \( \delta_1 \) and \( \delta_2 \) are constants.)
   
   True
   False

d. [3 points] If \( \lambda^2 + p\lambda + q = 0 \) is the characteristic equation of a constant-coefficient linear differential equation \( L[y] = g(t) \), then solving for \( Y(s) = \mathcal{L}\{y(t)\} \) will result in an expression involving a product of \( (s^2 + ps + q)^{-1} \) with other terms.
   
   True
   False

e. [3 points] If \( f(t) \neq 0 \) has Laplace transform \( \mathcal{L}\{f(t)\} = F(s) \) and \( g(t) = \begin{cases} f(t), & 0 < t < c, \\ 0, & t \geq c \end{cases} \), then \( \mathcal{L}\{g(t)\} = (1 - e^{-sc})F(s) \).
   
   True
   False