

5. [14 points] Consider the operators $T[y] = yy'' + 2y^2y'$ and $U[y] = t^2y'' - ty' - 3y$.
- a. [9 points] Show that T is nonlinear while U is linear.

- b. [5 points] Show that $y_1 = t^{-1}$ and $y_2 = t^3$ constitute a fundamental set of solutions to the equation $U[y] = 0$. What is the general solution to $U[y] = 0$?
(You may assume that $t > 0$.)