1. [15 points] Find explicit, real-valued solutions to each of the following, as indicated. For this problem, use Laplace transforms, **not** some other solution technique.

a. [8 points] $y'' + 2y' + y = e^{-t}$, $y(0) = 0$, $y'(0) = 1$.

Solution: Applying the forward transform, letting $Y = \mathcal{L}\{y\}$, we have $(s^2+2s+1)Y-1 = \frac{1}{s+1}$, so that

$$Y = \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}.$$

Note that, using rules (2) and (B) from the table, we have $\mathcal{L}^{-1}\{\frac{1}{(s+1)^2}\} = \mathcal{L}^{-1}\{-\frac{d}{ds}\frac{1}{(s+1)}\} = te^{-t}$, and $\mathcal{L}^{-1}\{\frac{1}{(s+1)^3}\} = \mathcal{L}^{-1}\{-\frac{1}{2}\frac{d}{ds}\frac{1}{(s+1)^2}\} = \frac{1}{2}t^2e^{-t}$. Thus,

$$\begin{aligned} y &= \mathcal{L}^{-1}\{\frac{1}{(s+1)^2}\} + \mathcal{L}^{-1}\{\frac{1}{(s+1)^3}\} \\ &= te^{-t} + \frac{1}{2}t^2e^{-t}. \end{aligned}$$

b. [7 points] $y'' + 6y' + 13y = 0$, $y(0) = 1$, $y'(0) = 0$.

Solution: Applying the forward transform, letting $Y = \mathcal{L}\{y\}$, we have $(s^2+6s+13)Y - s - 6 = 0$, so that

$$Y = \frac{s+3}{(s+3)^2+4} + \frac{3}{(s+3)^2+4}.$$

We can invert both of these with rules (4), (5), and (C). We have

$$\begin{aligned} y = \mathcal{L}^{-1}\{Y\} &= \mathcal{L}^{-1}\left\{\frac{s+3}{(s+3)^2+4} + \frac{3}{(s+3)^2+4}\right\} \\ &= e^{-3t}\cos(2t) + \frac{3}{2}e^{-3t}\sin(2t). \end{aligned}$$