4. [15 points] In lab 2 we considered the van der Pol oscillator, modeled by the equation, $x'' + \mu(x^2 - 1)x' + x = 0$. Recall that there is a single critical point for this system, $x = 0$, near which we may model the behavior of the oscillator with the linear equation $x'' - \mu x' + x = 0$.

a. [5 points] Suppose that $\mu = -1$. Find the amplitude of the solution to the linear problem with initial condition $x(0) = 2$, $x'(0) = 3$. What is the time after which this amplitude never exceeds some value a_0?

b. [5 points] Suppose we force the linear system with an oscillatory input, so that we are considering $x'' - \mu x' + x = \cos(\omega t)$ (and $\omega \neq 0$). For what values of μ will the system have an oscillatory steady-state solution with frequency ω?

c. [5 points] Suppose that, for some choice of μ, the system $x'' - \mu x' + x = \cos(\omega t)$ has an oscillatory steady-state solution, and that the gain function $G(\omega)$ for this solution is shown to the right, below. If the steady-state solution to the problem is $y_{ss} = R\cos(t - \pi/2)$, what are the R in the solution, and ω in the forcing term? Why?