4. [15 points] In lab 2 we considered the van der Pol oscillator, modeled by the equation, \( x'' + \mu(x^2 - 1)x' + x = 0 \). Recall that there is a single critical point for this system, \( x = 0 \), near which we may model the behavior of the oscillator with the linear equation \( x'' - \mu x' + x = 0 \).

a. [5 points] Suppose that \( \mu = -1 \). Find the amplitude of the solution to the linear problem with initial condition \( x(0) = 2, \ x'(0) = 3 \). What is the time after which this amplitude never exceeds some value \( a_0 \)?

b. [5 points] Suppose we force the linear system with an oscillatory input, so that we are considering \( x'' - \mu x' + x = \cos(\omega t) \) (and \( \omega \neq 0 \)). For what values of \( \mu \) will the system have an oscillatory steady-state solution with frequency \( \omega \)?

c. [5 points] Suppose that, for some choice of \( \mu \), the system \( x'' - \mu x' + x = \cos(\omega t) \) has an oscillatory steady-state solution, and that the gain function \( G(\omega) \) for this solution is shown to the right, below. If the steady-state solution to the problem is \( y_{ss} = R \cos(t - \pi/2) \), what are the \( R \) in the solution, and \( \omega \) in the forcing term? Why?