5. [14 points] In each of the following we consider a linear, second order, constant coefficient operator L, so that $L[y] = 0$ is a homogenous differential equation. (Note, however, that the operator L may be different in each of the parts below.) Let $y(0) = y_0$ and $y'(0) = v_0$, where y_0 and v_0 are real numbers.

a. [7 points] If the general solution to the equation $L[y] = 0$ is $y = c_1 e^{-3t}\cos(2t) + c_2 e^{-3t}\sin(2t)$, what is the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$?

b. [7 points] Now suppose that we are solving $L[y] = k$, for some constant k, and that y_0 and v_0 are both zero (so that $y(0) = y'(0) = 0$). If $Y(s) = \mathcal{L}\{y(t)\}$ is $Y = \frac{k}{s(s+3)(s+4)}$, what is the differential equation we are solving, and the general solution to the complementary homogeneous problem? Explain how you know your answer is correct.